

Simulating quantum circuits with non-Clifford noise

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Introduction

Program: IPAM RIPS

Research in Industrial Projects
for Students (RIPS)

Teams of 4 students with an
Academic Mentor (John) and
Industry Mentor(s) (Jeffrey
and Namit) work on Industry
Projects.



Background

The problem with classical simulations of quantum circuits

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\psi\rangle \otimes |\phi\rangle = \begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_4 \end{pmatrix}$$

$$|\psi\rangle \otimes |\phi\rangle \otimes |\gamma\rangle = \begin{pmatrix} a_1 \\ \vdots \\ \vdots \\ a_{16} \end{pmatrix}$$

- State grows with $O(2^n)$
- The current state of the art allows for the classical simulation of a 54-qubit system [4]

[4]]Pednault, Edwin P. D. et al. "Leveraging Secondary Storage to Simulate Deep 54-qubit Sycamore Circuits." arXiv: Quantum Physics (2019): n. pag.

Is there an effective classical simulation of a quantum circuit?

Gottesman-Knill theorem: A quantum computation can be efficiently simulated on a classical computer if the operations performed are limited to a set of operators called the Clifford group, which are generated by compositions of the following operators.

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

* This gate-set is not universal with respect to all possible quantum operations.

Stabilizer formalism

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Stab states are neat and can be *uniquely* represented by a **stabilizer group** defined by:

$$\text{Stab}(|\psi\rangle) = \{U \in C : U|\psi\rangle = |\psi\rangle\}$$

Stabilizer formalism

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$$\langle g_1, g_2, \dots, g_n \rangle \rightarrow \langle Cg_1C^\dagger, Cg_2C^\dagger, \dots, Cg_nC^\dagger \rangle$$

Our Problem

- Recall that the clifford group is *not* universal
- We are interested in simulating *non-clifford* noise channels *via* the stabilizer formalism.
 - Particularly **Krauss Channels**

Krauss Channels

A **Krauss Channel** is a completely positive, trace preserving map mapping density matrices to density matrices. If $\xi = \{K_i\}$ is a collection of Krauss operators K , then

$$\xi(\rho) = \sum_{i=1}^K K_i \rho K_i^\dagger$$

Quantum Trajectories

- When simulating quantum circuits, working with density matrices is *very* expensive.
- Quantum Trajectories allow us to work with pure states (kets) and achieve a similar outcome.

Quantum Trajectories

TODO: explain quantum trajectories

Q: How are quantum trajectories equivalent to OSR? How do pure states act as a good stand-in for mixed states (density matrices)?

Problem Statement

- Our problem is to apply these non-clifford Krauss channels *while staying within the stabilizer formalism*
- Doing so would allow the efficient simulation of non-clifford noise.

Roadmap

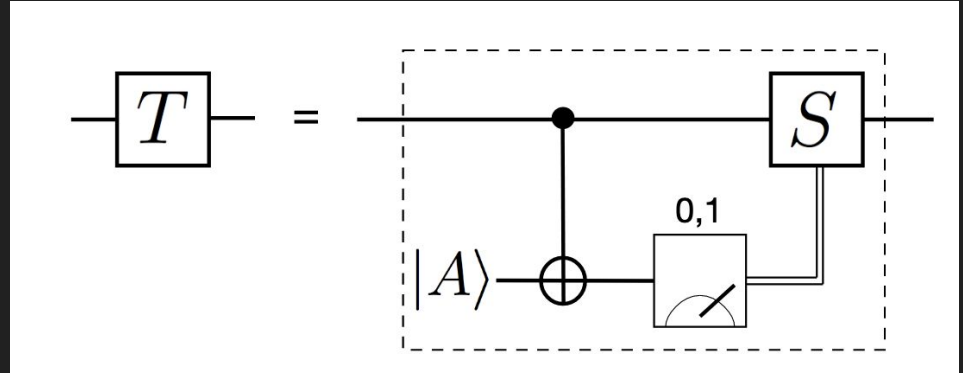
1. K-gadgets Approach
 - a. Bravyi's approach
 - b. Choi States

2. Dilation Approach
 - a. SteinSprings
 - b. S.z. Nagy

T-gates, K-gates, and Compressions

Bravyi, 2016

1. CHPT forms a universal gate set for unitary matrices.



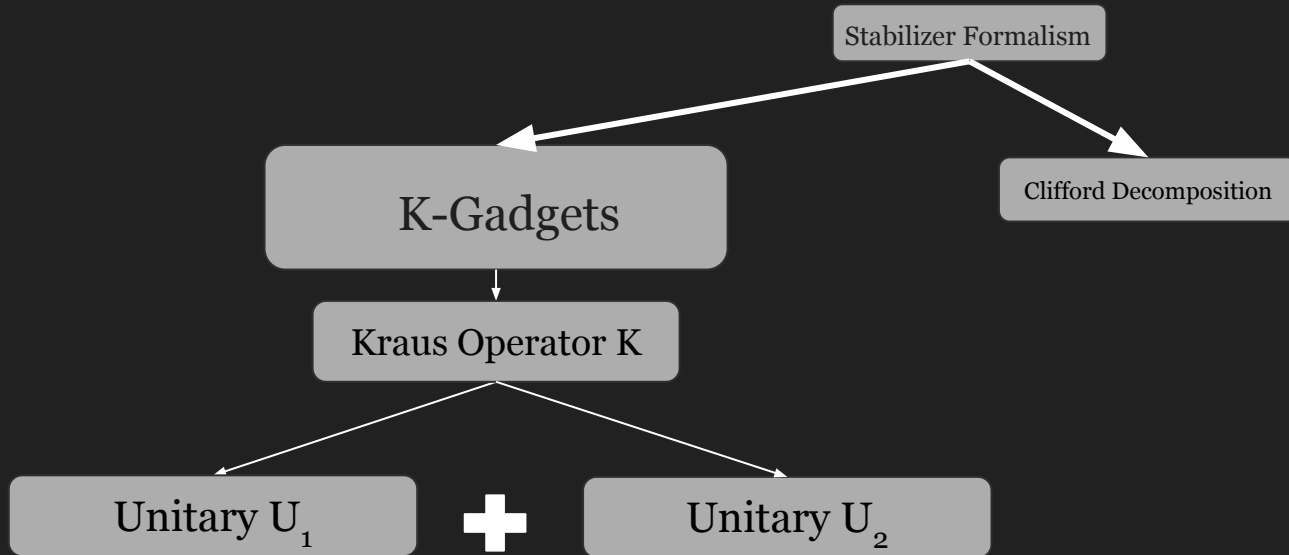
What we've done

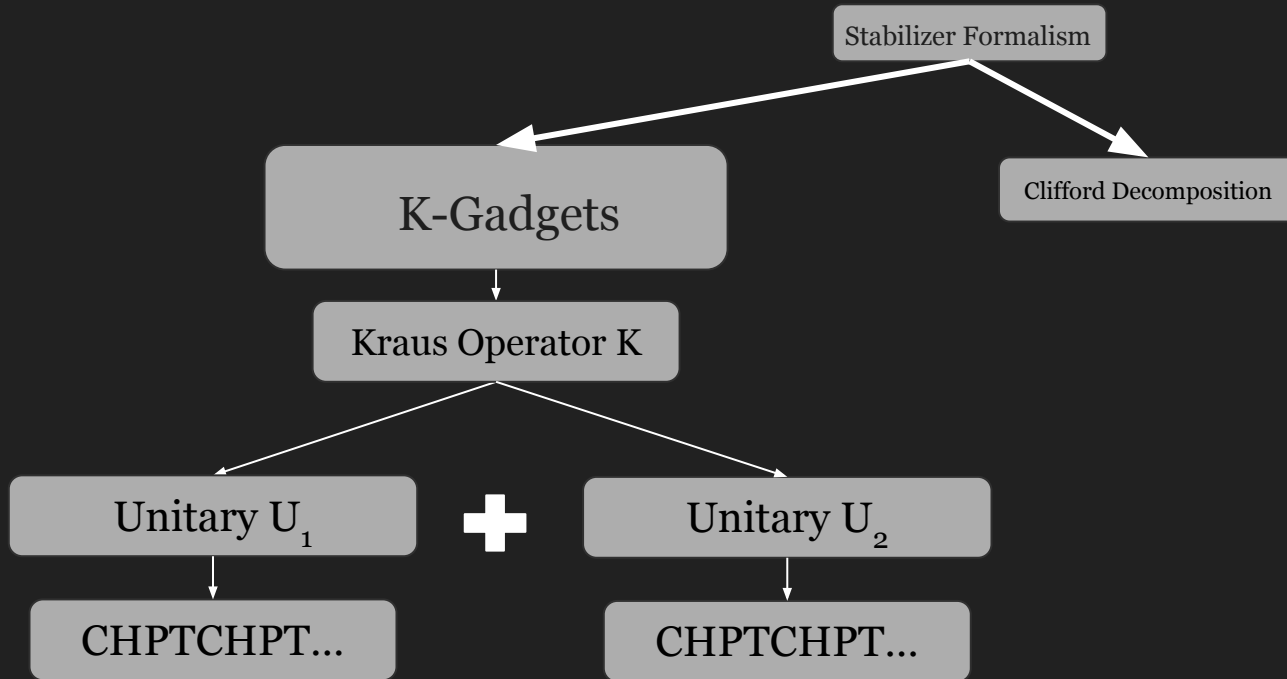
Stabilizer Formalism

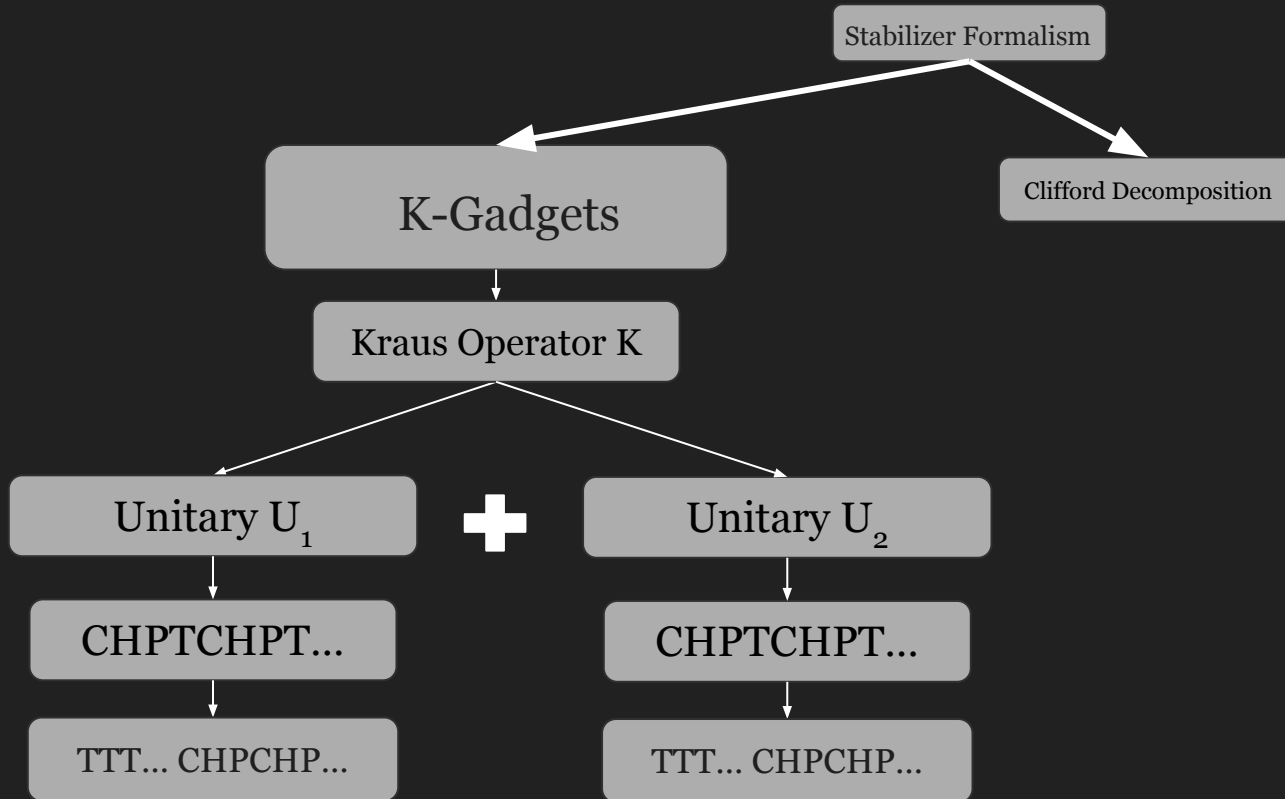
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graph TD; A[Stabilizer Formalism] --> B[K-Gadgets]; A --> C[Clifford Decomposition];
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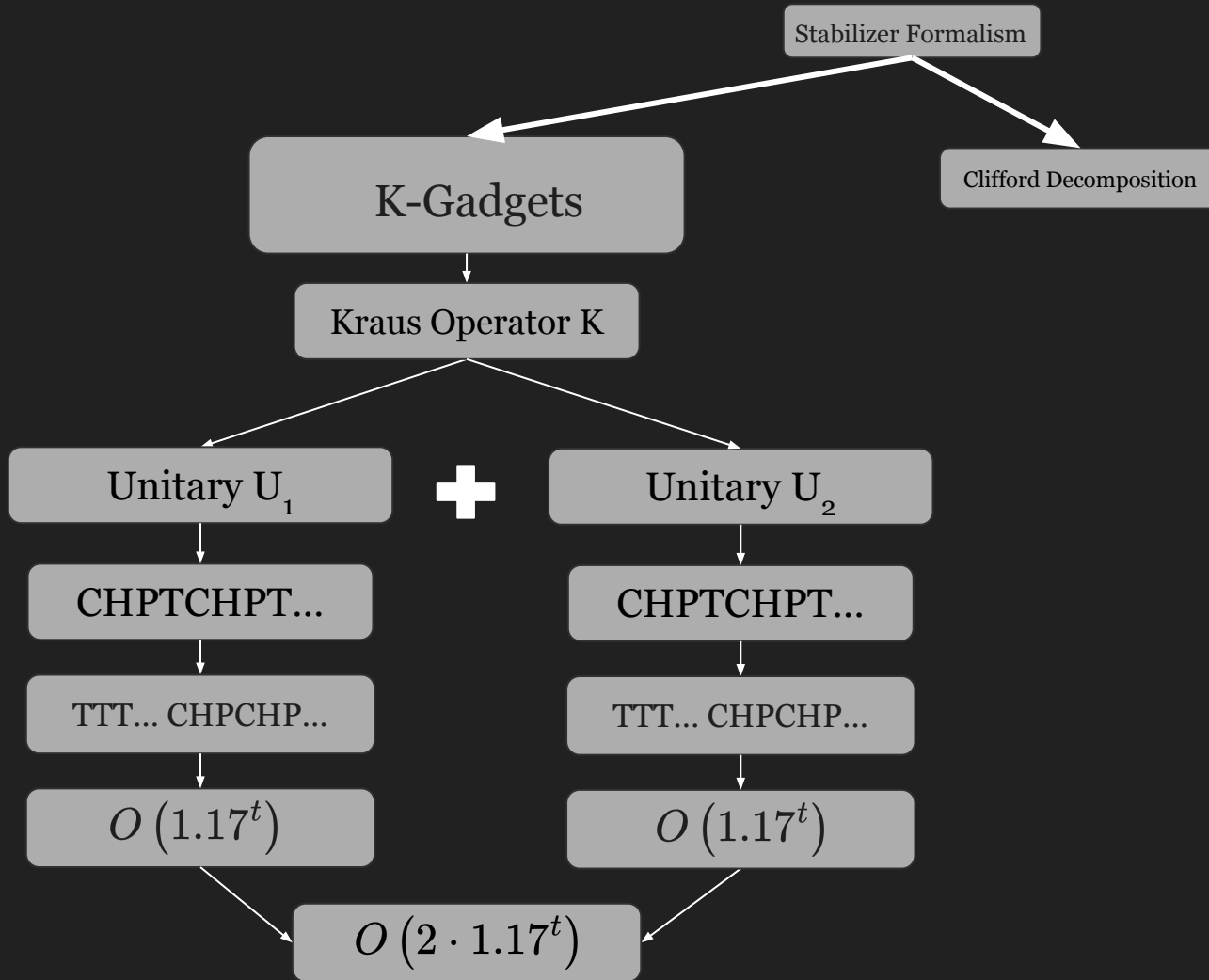
K-Gadgets

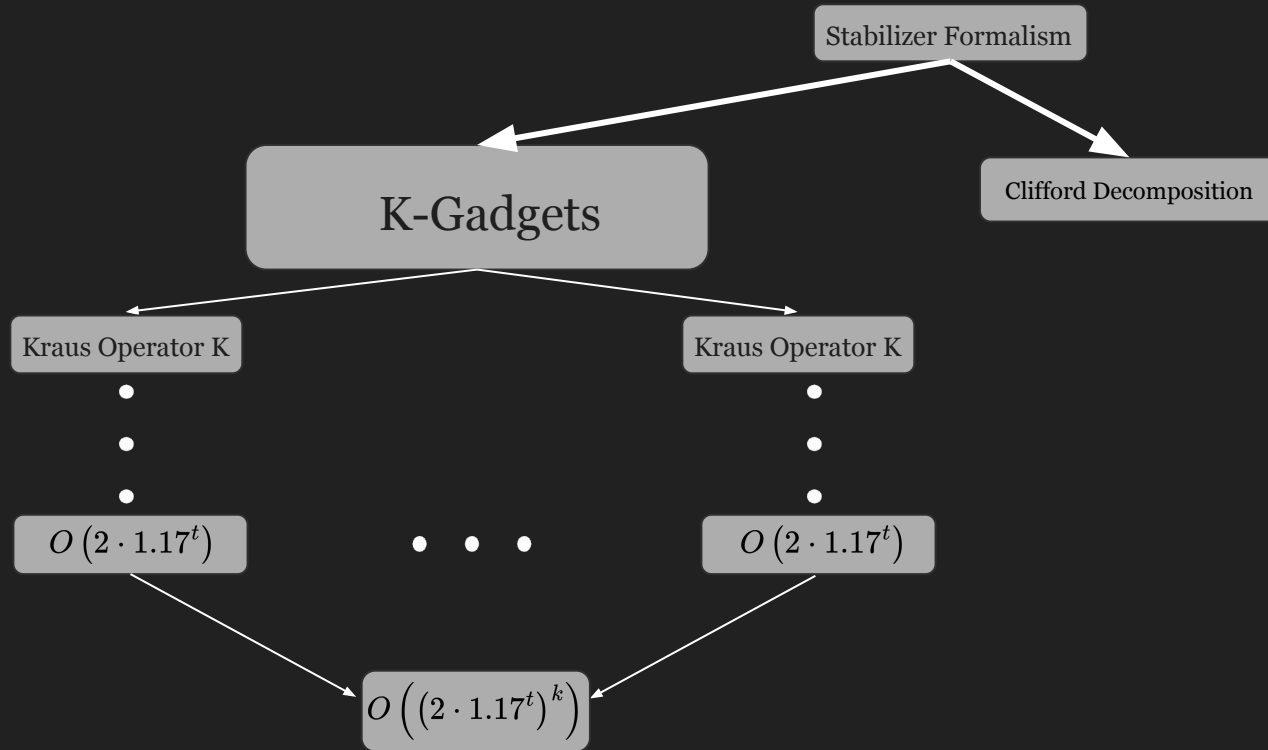
Clifford Decomposition

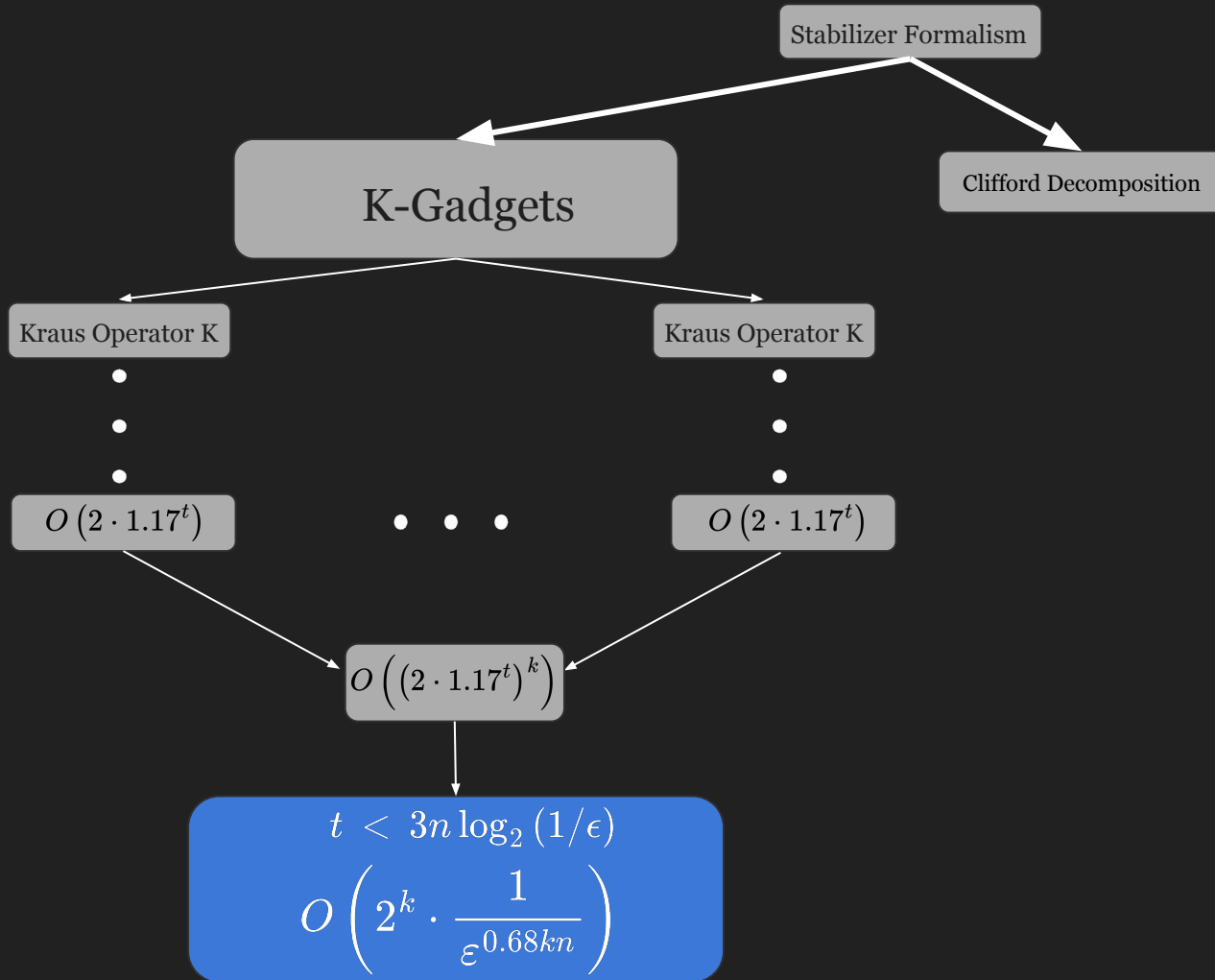






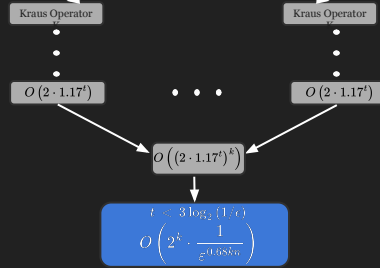






Stabilizer Formalism

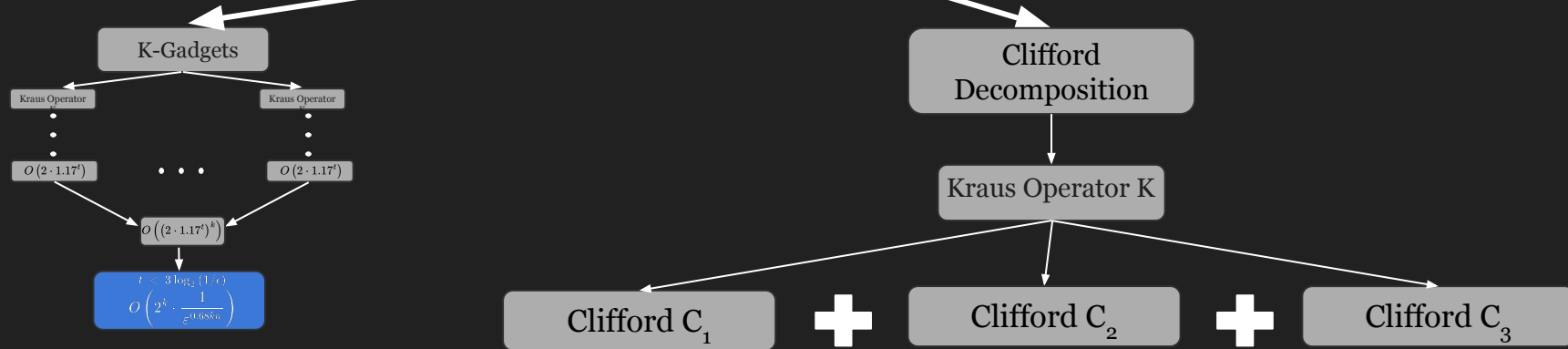
K-Gadgets



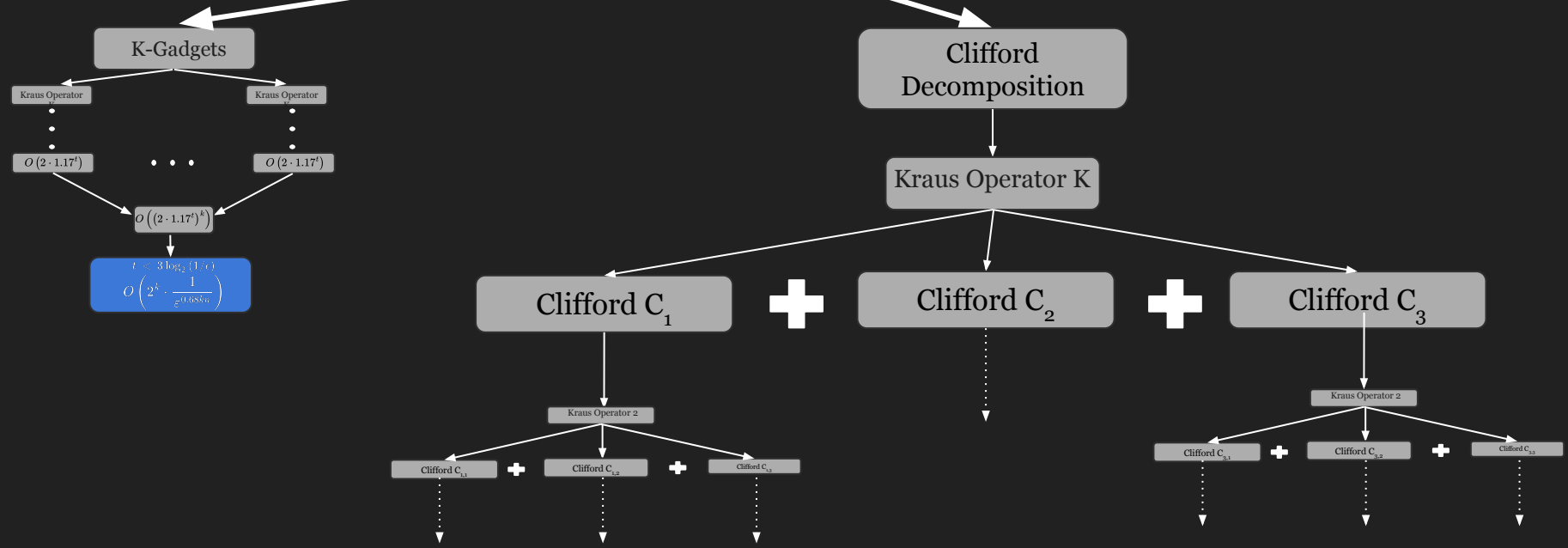
Clifford Decomposition

Kraus Operator K

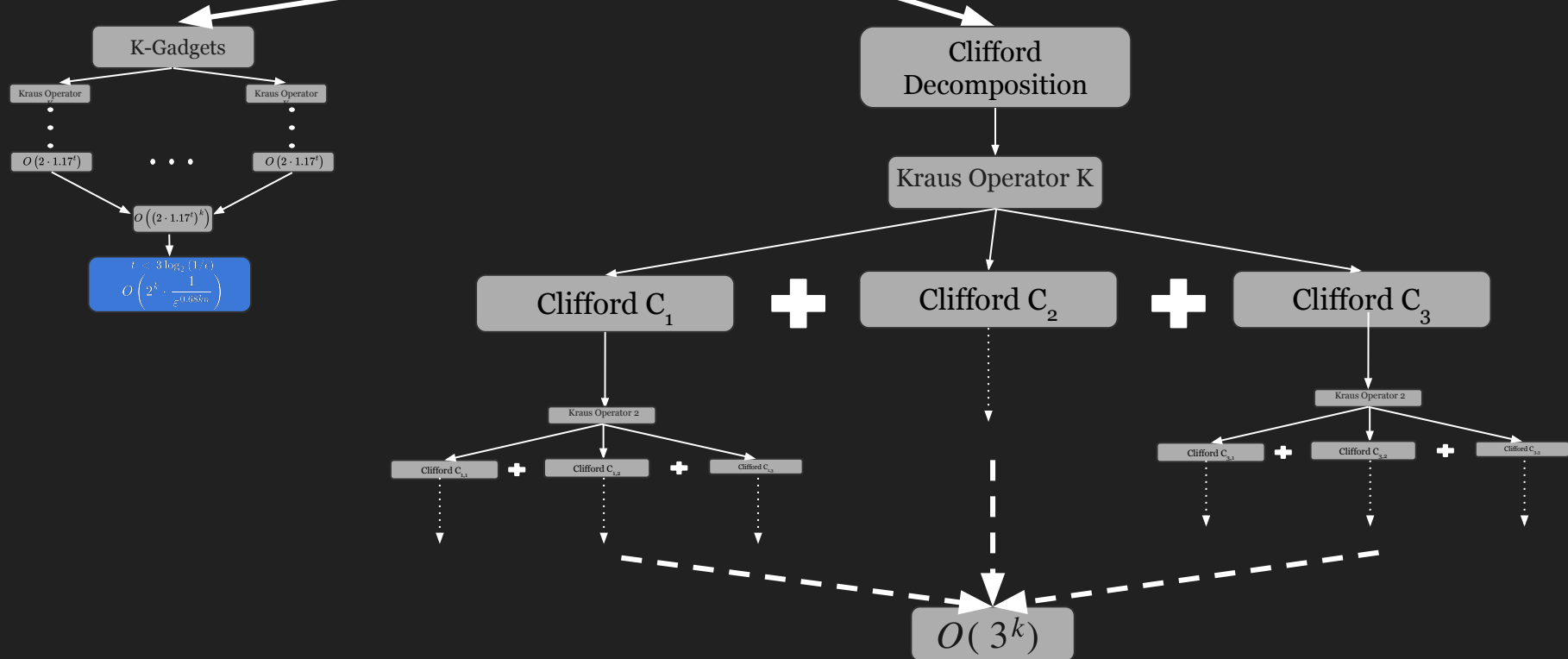
Stabilizer Formalism



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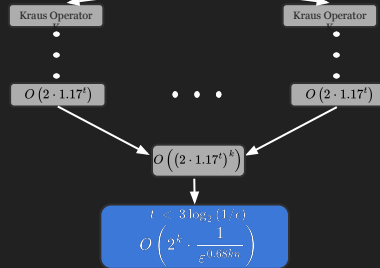


Stabilizer Formalism



Stabilizer Formalism

K-Gadgets



Clifford
Decomposition

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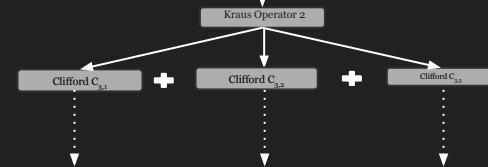
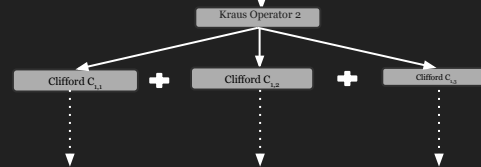
Clifford C_1

+

Clifford C_2

+

Clifford C_3



$O(3^k)$

$O\left(\prod_{i=1}^k R_i\right)$

K-Gadgets



$$O\left(2^k \cdot \frac{1}{\varepsilon^{0.68kn}}\right)$$

vs

Clifford
Decomposition



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$$|C_n| \approx 4^{(n^2+n)}$$

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$$|C_n| \approx 4^{(n^2+n)}$$

$$|C_1| = 24, |C_2| \approx 11000, |C_3| \approx 93,000,000$$

Future directions

Introducing Clifford decompositions

Overarching question for project: How can we efficiently simulate quantum computation on classical computers?

Recall: By the Gottesman-Knill, there is a certain subset of quantum operations, called the Clifford group, which can be simulated efficiently on classical computers.

[Well, Clifford groups are closed under composition, but what about linear sums?]

- Obtaining optimal Clifford decompositions was tasked to us by NASA Ames, used as a non-constructive method in some recent influential papers [A], and identified as a future improvement by some quantum simulation libraries [B].

[A] Bravyi, Sergey, Dan Browne, Padraic Calpin, Earl Campbell, David Gosset, and Mark Howard. "Simulation of Quantum Circuits by Low-Rank Stabilizer Decompositions." *Quantum* 3 (September 2, 2019): 181. DOI: 10.22331/q-2019-09-02-181.

[B]: Garcia, Hector J., and Igor L. Markov. "Simulation of Quantum Circuits via Stabilizer Frames." *IEEE Transactions on Computers* 64, no. 8 (August 1, 2015): 2323–36. DOI: 10.1109/TC.2014.2360532.

The system and the environment

We can represent a qubit with a matrix, called a **density matrix** ρ

$$\rho_{net} = \rho_{sys} \otimes \rho_{env}$$

